3.1 Functions

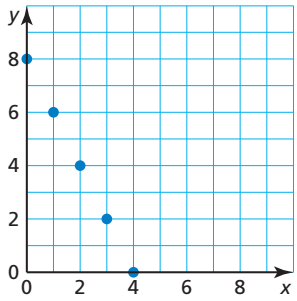
Essential Question What is a function?

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the x-coordinates are inputs and the y-coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

EXPLORATION 1 Describing a Function

Work with a partner. Functions can be described in many ways.

- by an equation
- by an input-output table
- using words
- by a graph
- as a set of ordered pairs



a. Explain why the graph shown represents a function.

b. Describe the function in two other ways.

ANALYZING RELATIONSHIPS

To be proficient in math, you need to analyze relationships mathematically to draw conclusions.

EXPLORATION 2 Identifying Functions

Work with a partner. Determine whether each relation represents a function. Explain your reasoning.

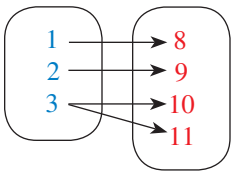
a.

Input, x	0	1	2	3	4
Output, y	8	8	8	8	8

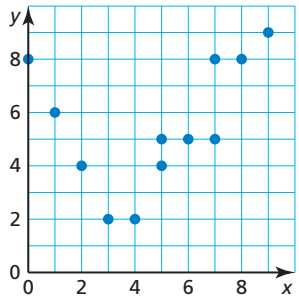
b.

Input, x	8	8	8	8	8
Output, y	0	1	2	3	4

c.



d.



e. $(-2, 5), (-1, 8), (0, 6), (1, 6), (2, 7)$

f. $(-2, 0), (-1, 0), (-1, 1), (0, 1), (1, 2), (2, 2)$

g. Each radio frequency x in a listening area has exactly one radio station y.

h. The same television station x can be found on more than one channel y.

i. $x = 2$

j. $y = 2x + 3$

Communicate Your Answer

3. What is a function? Give examples of relations, other than those in Explorations 1 and 2, that (a) are functions and (b) are not functions.

3.1 Lesson

Core Vocabulary

relation, *p. 104*
function, *p. 104*
domain, *p. 106*
range, *p. 106*
independent variable, *p. 107*
dependent variable, *p. 107*

Previous
ordered pair
mapping diagram

What You Will Learn

- Determine whether relations are functions.
- Find the domain and range of a function.
- Identify the independent and dependent variables of functions.

Determining Whether Relations Are Functions

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the x-coordinates are inputs and the y-coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

EXAMPLE 1 Determining Whether Relations Are Functions

Determine whether each relation is a function. Explain.

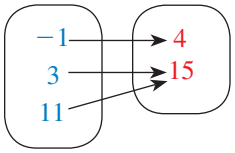
a. $(-2, 2), (-1, 2), (0, 2), (1, 0), (2, 0)$

b. $(4, 0), (8, 7), (6, 4), (4, 3), (5, 2)$

c.

Input, x	-2	-1	0	0	1	2
Output, y	3	4	5	6	7	8

d. Input, x Output, y



REMEMBER
A relation can be represented by a mapping diagram.

SOLUTION

a. Every input has exactly one output.

▶ So, the relation is a function.

b. The input 4 has two outputs, 0 and 3.

▶ So, the relation is *not* a function.

c. The input 0 has two outputs, 5 and 6.

▶ So, the relation is *not* a function.

d. Every input has exactly one output.

▶ So, the relation is a function.

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Determine whether the relation is a function. Explain.

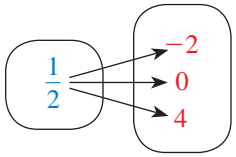
1. $(-5, 0), (0, 0), (5, 0), (5, 10)$

2. $(-4, 8), (-1, 2), (2, -4), (5, -10)$

3.

Input, x	Output, y
2	2.6
4	5.2
6	7.8

4. Input, x Output, y

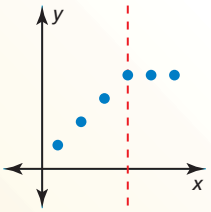


Core Concept

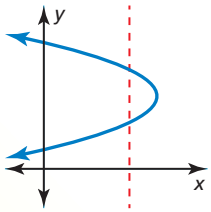
Vertical Line Test

Words A graph represents a function when no vertical line passes through more than one point on the graph.

Examples Function



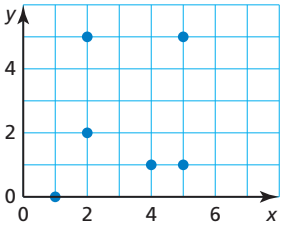
Not a function



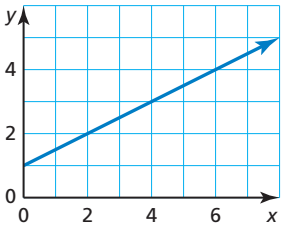
EXAMPLE 2 Using the Vertical Line Test

Determine whether each graph represents a function. Explain.

a.



b.



SOLUTION

a. You can draw a vertical line through (2, 2) and (2, 5).

▶ So, the graph does *not* represent a function.

b. No vertical line can be drawn through more than one point on the graph.

▶ So, the graph represents a function.

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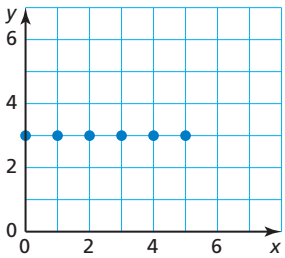
Determine whether the graph represents a function. Explain.

5.



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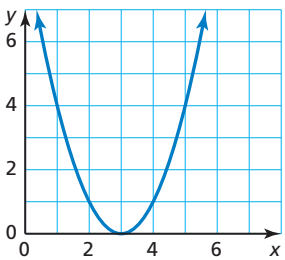

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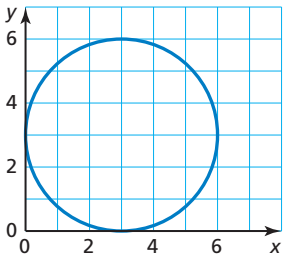

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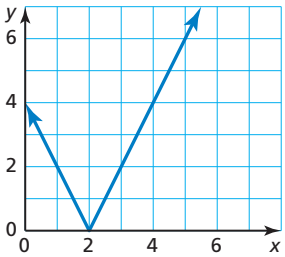


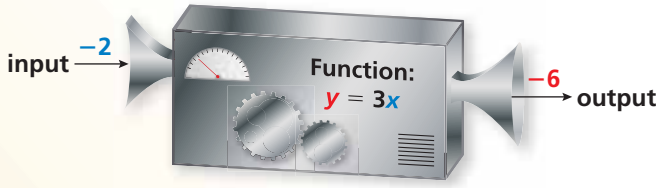
Finding the Domain and Range of a Function

Core Concept

The Domain and Range of a Function

The **domain** of a function is the set of all possible input values.

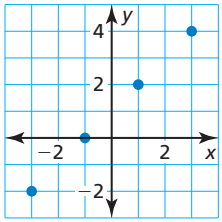
The **range** of a function is the set of all possible output values.



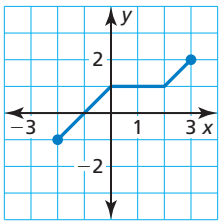
EXAMPLE 3 Finding the Domain and Range from a Graph

Find the domain and range of the function represented by the graph.

a.

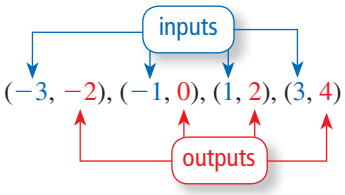


b.



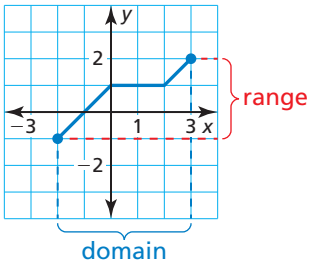
SOLUTION

a. Write the ordered pairs. Identify the inputs and outputs.



▶ The domain is −3, −1, 1, and 3.
The range is −2, 0, 2, and 4.

b. Identify the x- and y-values represented by the graph.



▶ The domain is $-2 \le x \le 3$.
The range is $-1 \le y \le 2$.

STUDY TIP

A relation also has a domain and a range.

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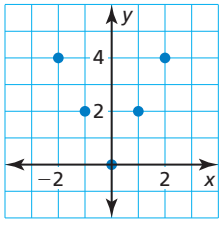
Find the domain and range of the function represented by the graph.

9.



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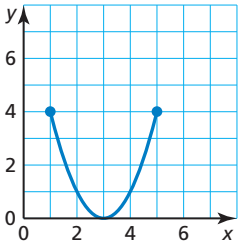

Identifying Independent and Dependent Variables

The variable that represents the input values of a function is the **independent variable** because it can be *any* value in the domain. The variable that represents the output values of a function is the **dependent variable** because it *depends* on the value of the independent variable. When an equation represents a function, the dependent variable is defined in terms of the independent variable. The statement "y is a function of x" means that y varies depending on the value of x.

$$y = -x + 10$$

dependent variable, y — independent variable, x

EXAMPLE 4 Identifying Independent and Dependent Variables

The function $y = -3x + 12$ represents the amount y (in fluid ounces) of juice remaining in a bottle after you take x gulps.

a. Identify the independent and dependent variables.

b. The domain is 0, 1, 2, 3, and 4. What is the range?

SOLUTION

a. The amount y of juice remaining depends on the number x of gulps.

▶ So, y is the dependent variable, and x is the independent variable.

b. Make an input-output table to find the range.

Input, x	$-3x + 12$	Output, y
0	$-3(0) + 12$	12
1	$-3(1) + 12$	9
2	$-3(2) + 12$	6
3	$-3(3) + 12$	3
4	$-3(4) + 12$	0

▶ The range is 12, 9, 6, 3, and 0.

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11. The function $a = -4b + 14$ represents the number a of avocados you have left after making b batches of guacamole.

a. Identify the independent and dependent variables.

b. The domain is 0, 1, 2, and 3. What is the range?

12. The function $t = 19m + 65$ represents the temperature t (in degrees Fahrenheit) of an oven after preheating for m minutes.

a. Identify the independent and dependent variables.

b. A recipe calls for an oven temperature of 350°F. Describe the domain and range of the function.

3.1 Exercises

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Vocabulary and Core Concept Check

1. WRITING How are independent variables and dependent variables different?

2. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

Find the range of the function represented by the table.

Find the inputs of the function represented by the table.

x	-1	0	1
y	7	5	-1

Find the x-values of the function represented by $(-1, 7)$, $(0, 5)$, and $(1, -1)$.

Find the domain of the function represented by $(-1, 7)$, $(0, 5)$, and $(1, -1)$.

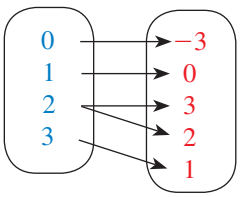
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, determine whether the relation is a function. Explain. *(See Example 1.)*

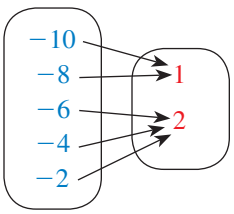
3. $(1, -2)$, $(2, 1)$, $(3, 6)$, $(4, 13)$, $(5, 22)$

4. $(7, 4)$, $(5, -1)$, $(3, -8)$, $(1, -5)$, $(3, 6)$

5. Input, x Output, y



6. Input, x Output, y



7.

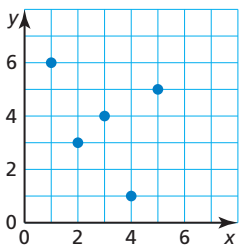
Input, x	16	1	0	1	16
Output, y	-2	-1	0	1	2

8.

Input, x	-3	0	3	6	9
Output, y	11	5	-1	-7	-13

In Exercises 9–12, determine whether the graph represents a function. Explain. *(See Example 2.)*

9.



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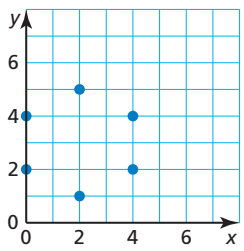

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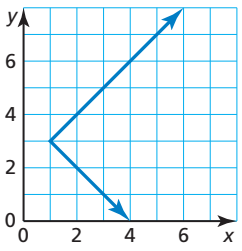

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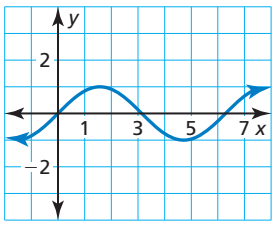
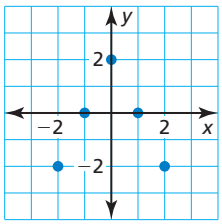


In Exercises 13–16, find the domain and range of the function represented by the graph. *(See Example 3.)*

13.



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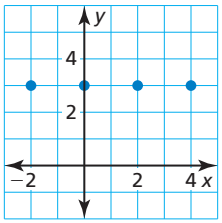

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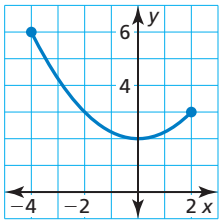

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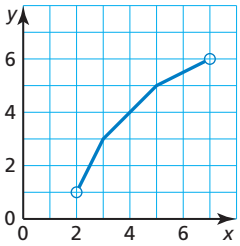


17. MODELING WITH MATHEMATICS The function $y = 25x + 500$ represents your monthly rent y (in dollars) when you pay x days late. *(See Example 4.)*

a. Identify the independent and dependent variables.

b. The domain is 0, 1, 2, 3, 4, and 5. What is the range?

18. MODELING WITH MATHEMATICS The function $y = 3.5x + 2.8$ represents the cost y (in dollars) of a taxi ride of x miles.

a. Identify the independent and dependent variables.

b. You have enough money to travel at most 20 miles in the taxi. Find the domain and range of the function.

ERROR ANALYSIS In Exercises 19 and 20, describe and correct the error in the statement about the relation shown in the table.

Input, x	1	2	3	4	5
Output, y	6	7	8	6	9

19. ✗ The relation is *not* a function. One output is paired with two inputs.

20. ✗ The relation is a function. The range is 1, 2, 3, 4, and 5.

ANALYZING RELATIONSHIPS In Exercises 21 and 22, identify the independent and dependent variables.

21. The number of quarters you put into a parking meter affects the amount of time you have on the meter.

22. The battery power remaining on your MP3 player is based on the amount of time you listen to it.

23. MULTIPLE REPRESENTATIONS The balance y (in dollars) of your savings account is a function of the month x.

Month, x	0	1	2	3	4
Balance (dollars), y	100	125	150	175	200

a. Describe this situation in words.

b. Write the function as a set of ordered pairs.

c. Plot the ordered pairs in a coordinate plane.

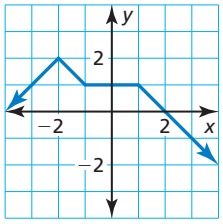
24. MULTIPLE REPRESENTATIONS The function $1.5x + 0.5y = 12$ represents the number of hardcover books x and softcover books y you can buy at a used book sale.

a. Solve the equation for y.

b. Make an input-output table to find ordered pairs for the function.

c. Plot the ordered pairs in a coordinate plane.

25. ATTENDING TO PRECISION The graph represents a function. Find the input value corresponding to an output of 2.



26. OPEN-ENDED Fill in the table so that when t is the independent variable, the relation is a function, and when t is the dependent variable, the relation is not a function.

t				
v				

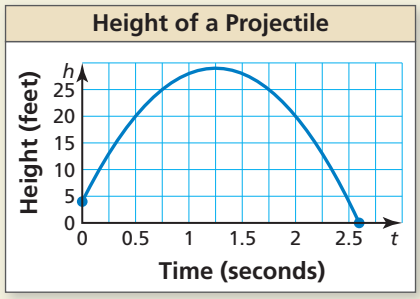
27. ANALYZING RELATIONSHIPS You select items in a vending machine by pressing one letter and then one number.

a. Explain why the relation that pairs letter-number combinations with food or drink items is a function.

b. Identify the independent and dependent variables.

c. Find the domain and range of the function.

28. HOW DO YOU SEE IT? The graph represents the height h of a projectile after t seconds.



a. Explain why h is a function of t.

b. Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

c. Approximate the domain of the function.

d. Is t a function of h? Explain.

29. MAKING AN ARGUMENT Your friend says that a line always represents a function. Is your friend correct? Explain.

30. THOUGHT PROVOKING Write a function in which the inputs and/or the outputs are not numbers. Identify the independent and dependent variables. Then find the domain and range of the function.

ATTENDING TO PRECISION In Exercises 31–34, determine whether the statement uses the word *function* in a way that is mathematically correct. Explain your reasoning.

31. The selling price of an item is a function of the cost of making the item.

32. The sales tax on a purchased item in a given state is a function of the selling price.

33. A function pairs each student in your school with a homeroom teacher.

34. A function pairs each chaperone on a school trip with 10 students.

REASONING In Exercises 35–38, tell whether the statement is true or false. If it is false, explain why.

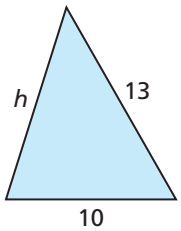
35. Every function is a relation.

36. Every relation is a function.

37. When you switch the inputs and outputs of any function, the resulting relation is a function.

38. When the domain of a function has an infinite number of values, the range always has an infinite number of values.

39. MATHEMATICAL CONNECTIONS Consider the triangle shown.



a. Write a function that represents the perimeter of the triangle.

b. Identify the independent and dependent variables.

c. Describe the domain and range of the function. (*Hint:* The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.)

REASONING In Exercises 40–43, find the domain and range of the function.

40. $y = |x|$

41. $y = -|x|$

42. $y = |x| - 6$

43. $y = 4 - |x|$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. *(Section 2.1)*

44. A number y is less than 16.

45. Three is no less than a number x.

46. Seven is at most the quotient of a number d and -5.

47. The sum of a number w and 4 is more than -12.

Evaluate the expression. *(Skills Review Handbook)*

48. 11^2

49. $(-3)^4$

50. -5^2

51. 2^5